Behavioral Dynamics Approach to Collective Crowd Behavior

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Bottom-up approach
Experiment-driven
Vision-based model

crowd simulations

Collective Behavior
(global pattern)

neighborhood model

interactions generate

visual neighborhood

positive feedback

Individual Behavior
(local coupling)

pedestrian model

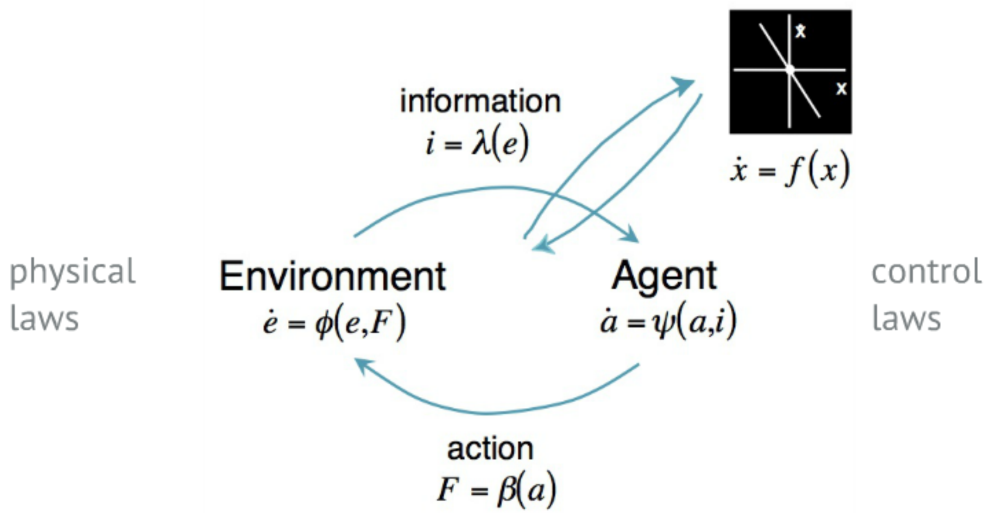
Haken (1977)

Claims

- Bottom-up, experimentally-driven model of crowd dynamics
- Accounts for both local trajectories and global motion
- Collective motion emerges from experimentally-specified local interactions

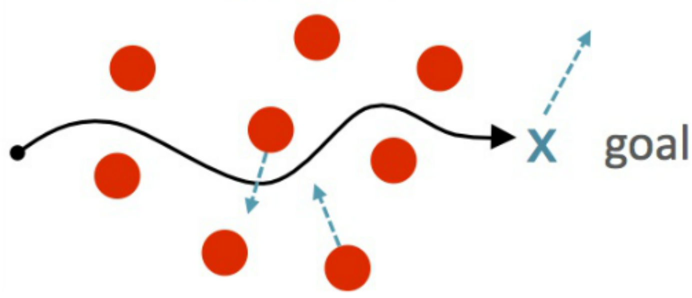
Next

Behavioral Dynamics Framework



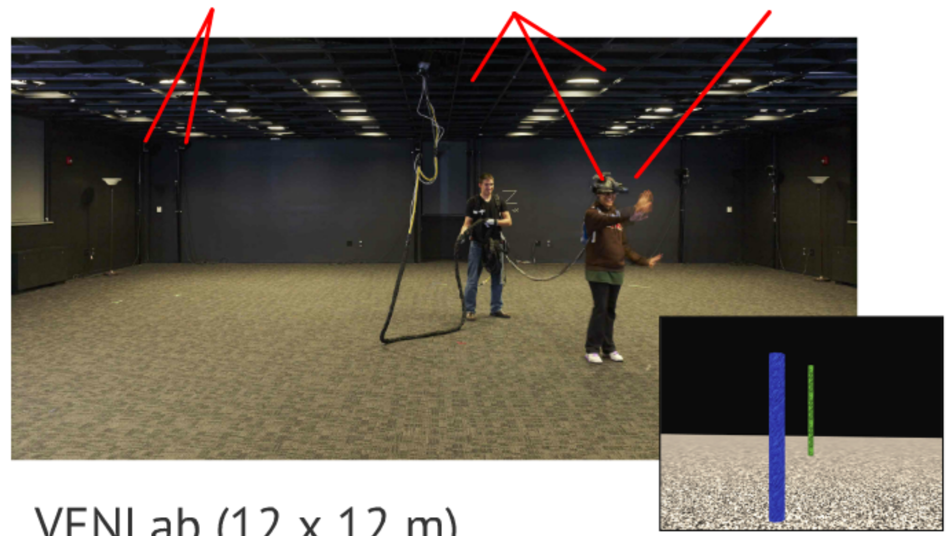
- Coupled dynamical systems
- On-line control, emergent behavior
- First pass: Behavioral model
- Second pass: Visual control laws

Our story thus far...



Elementary behaviors

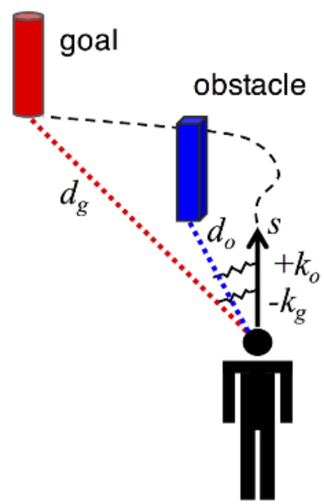
1. Goal
2. Obstacle
3. Moving target
4. Moving obstacle
5. Following...

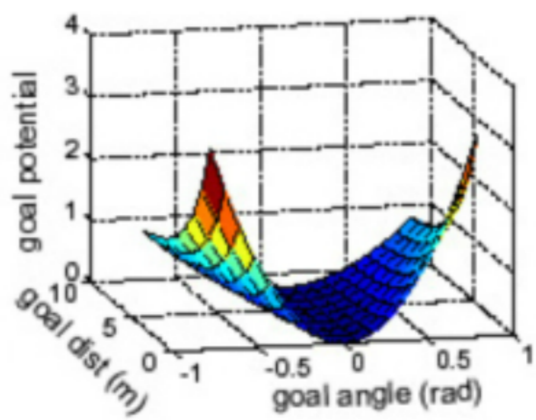


VENLab (12 x 12 m)

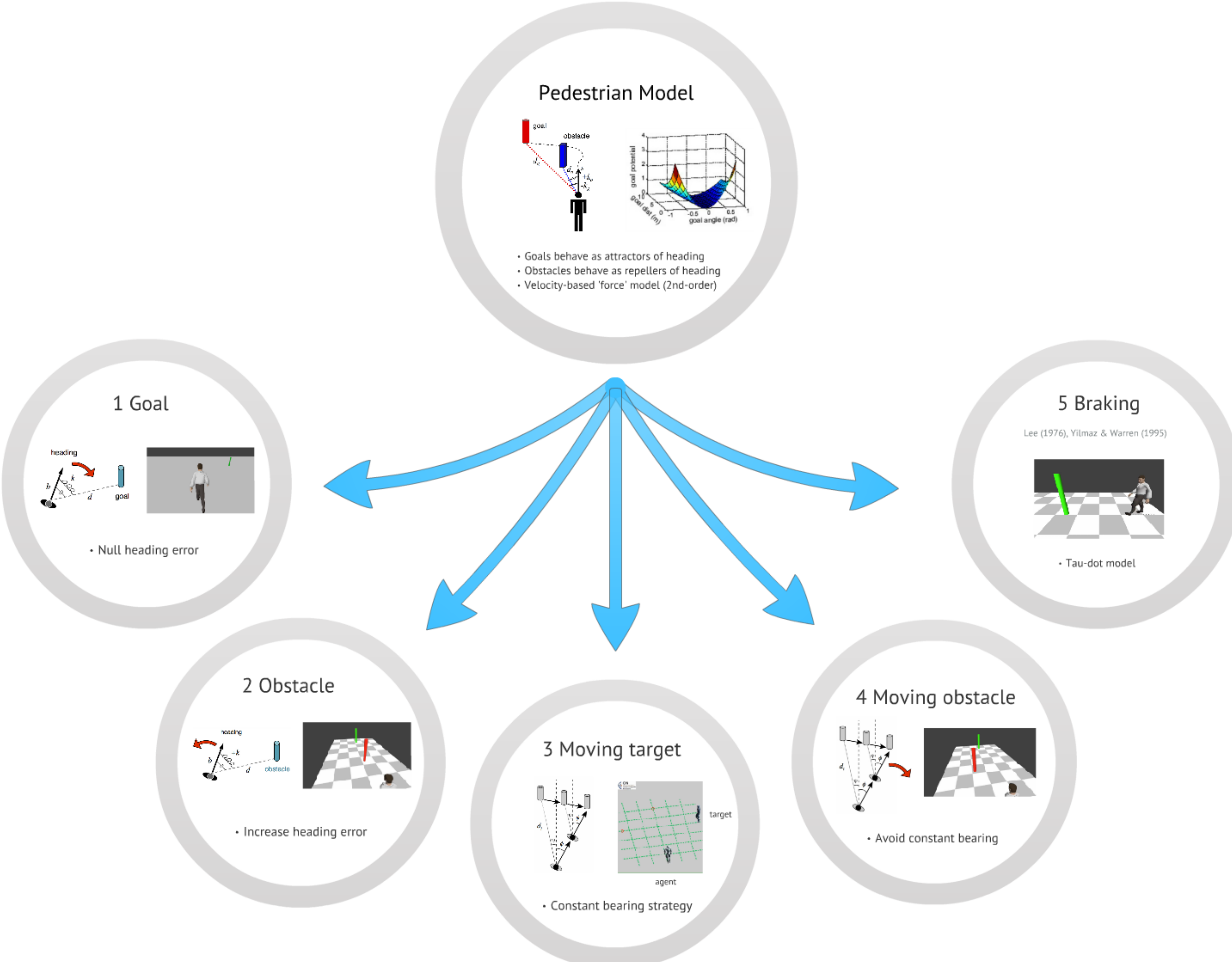
- Study each basic behavior
- Model as a nonlinear DS
- Combine components to model complex situations

Pedestrian Model



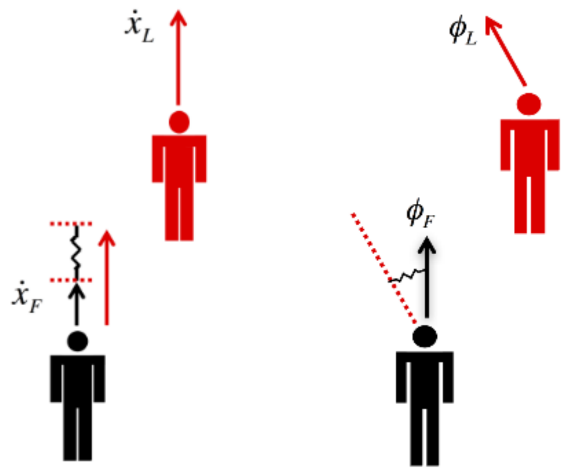


- Goals behave as attractors of heading
- Obstacles behave as repellers of heading
- Velocity-based 'force' model (2nd-order)


Pedestrian Model
goal
obstacle
• Goals behave as attractors of heading
• Obstacles behave as repellers of heading
• Velocity-based 'force' model (2nd-order)
goal potential
goal dist (m)
goal angle (rad)
1 Goal
heading
goal
• Null heading error
2 Obstacle
heading
obstacle
• Increase heading error
3 Moving target
target
agent
• Constant bearing strategy
4 Moving obstacle
• Avoid constant bearing
5 Braking
Lee (1976), Yilmaz & Warren (1995)
• Tau-dot model

Binary Interactions: Following

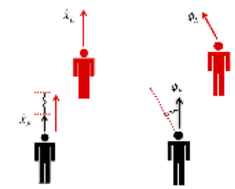
Alignment dynamics



- How do neighbors coordinate their speed and heading?
- Local coupling

Binary Interactions: Following

Alignment dynamics



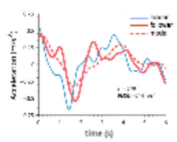
- How do neighbors coordinate their speed and heading?
- Local coupling

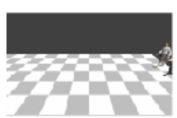
Conclusion 1: Alignment Dynamics

- Successfully model binary interactions as simple DS
- Vision model fits *better*
- Extend to multiple interactions in crowd?

6 Speed

Rio, Rhea & Warren (2014)





- Leader changes speed

mean x-r = 0.68
delay = 420ms

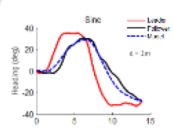
$$\ddot{x}_F = c(\dot{x}_L - \dot{x}_F)$$

- Compared 6 models
- Follower matches the leader's speed

mean r = 0.67

7 Heading

Dachner & Warren (2014)



- Leader turns twice

mean x-r = 0.92
delay = 984 ms

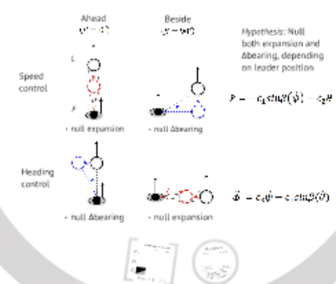
$$\ddot{\varphi}_F = -c\sin(\varphi_L - \varphi_F)$$

- Compared 4 models
- Follower matches the leader's heading

mean r = 0.72

Visual Control Laws for Following

Dachner & Warren (2017)



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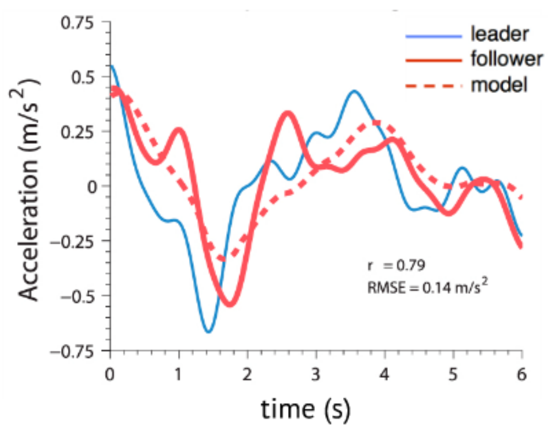

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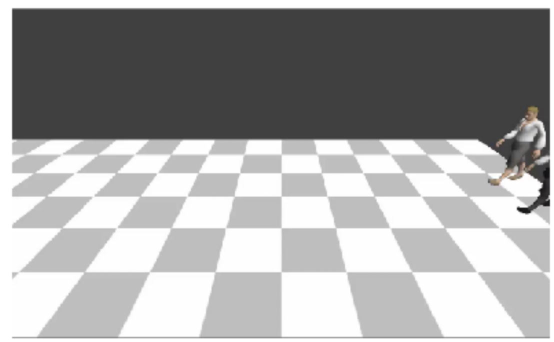

$$\ddot{x}_F = c\left(\dot{x}_L - \dot{x}_F\right)$$

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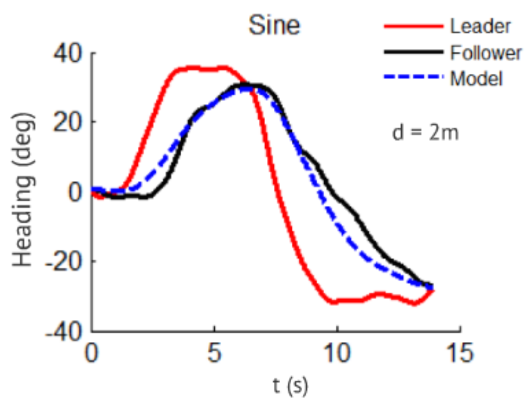
Dachner & Warren (2014)



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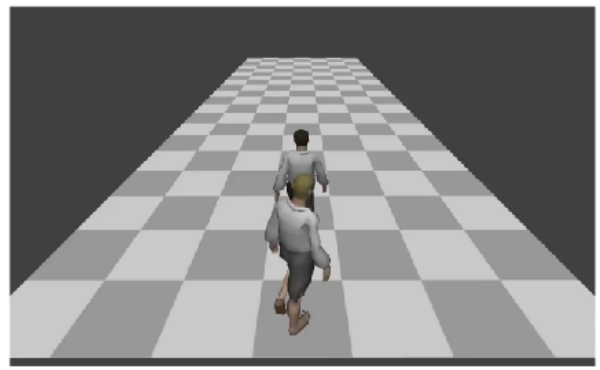


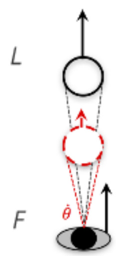
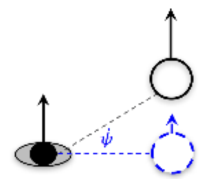
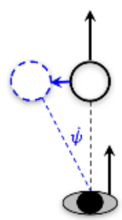
$$\ddot{\phi}_F = -c\sin(\phi_L - \phi_F)$$

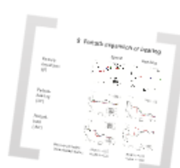
- Compared 4 models
- Follower matches the leader's heading

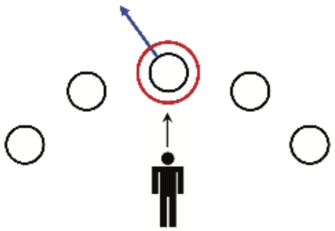
mean r = 0.72

Visual Control Laws for Following

Dachner & Warren (2017)

	Ahead ($\beta = 0^\circ$)	Beside ($\beta = 90^\circ$)	*Hypothesis:* Null both expansion and Δbearing, depending on leader position
Speed control	 • null expansion	 • null Δbearing	$\ddot{x} = -c_1 sin\beta(\dot{\psi}) - c_2\dot{\theta}$
Heading control	 • null Δbearing	 • null expansion	$\ddot{\phi} = c_3\dot{\psi} - c_4 sin\beta(\dot{\theta})$

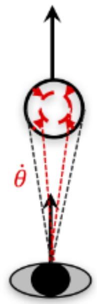


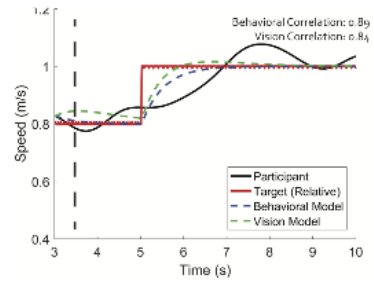


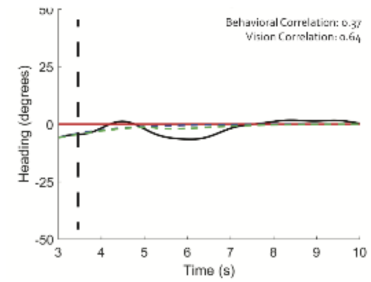
8 Perturb expansion or bearing

Speed | Heading

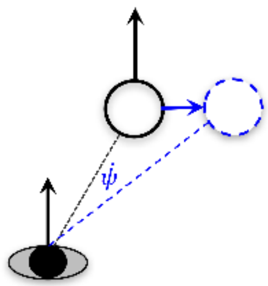
Perturb expansion (0°)

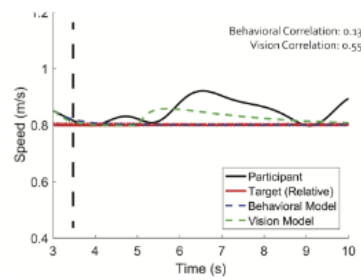


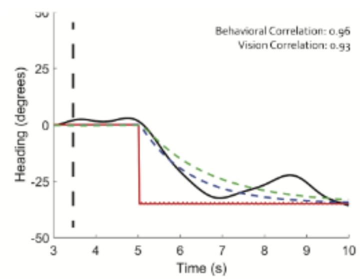




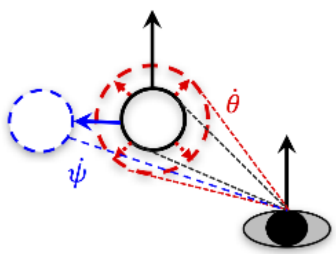
Perturb bearing (30°)

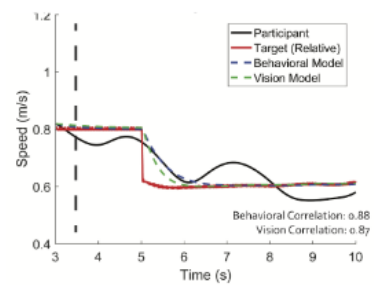


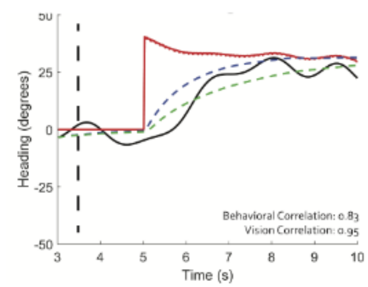




Perturb both (-60°)



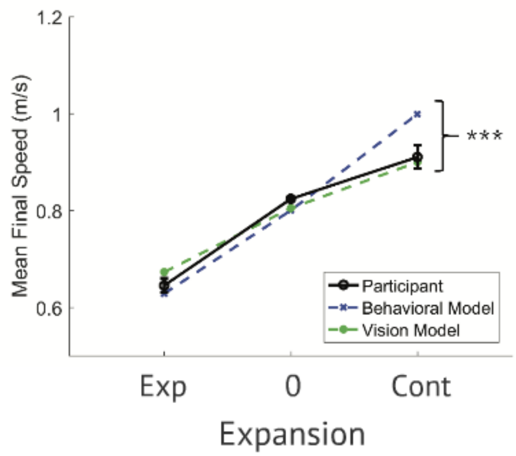




	Speed	Heading
Behavioral model:	mean r = 0.5	mean r = 0.8
Vision-based model:	mean r = 0.6	mean r = 0.9

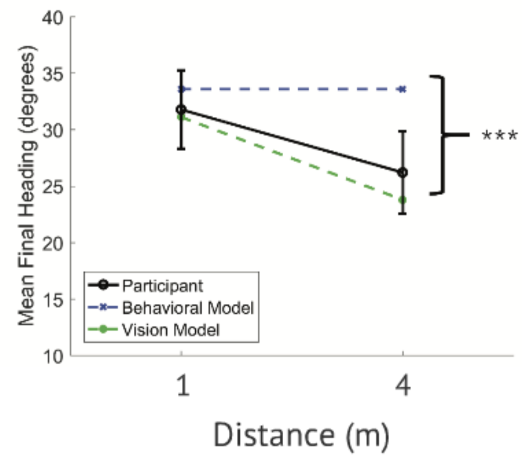
Modeling Results

Final Speed



- Asymmetry in response to expansion

Final Heading



- Decreased response with distance

- Vision model fits human data better than behavioral model ($p<.001$)
- Jointly null expansion and Δbearing, depending on leader position

Bottom-up approach
Experiment-driven
Vision-based model

crowd
simulations

Collective Behavior

(global pattern)

neighborhood
model

interactions
generate

visual
neighborhood

positive
feedback

Individual Behavior

(local coupling)

pedestrian
model

Claims

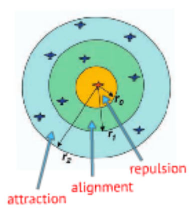
- Bottom-up, experimentally-driven model of crowd dynamics
- Accounts for both local trajectories and global motion
- Collective motion emerges from experimentally-specified local interactions

Haken (1977)

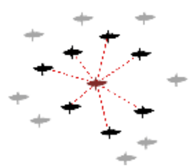
Next

Local Neighborhood

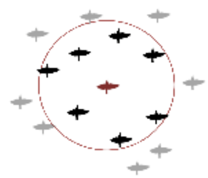
- How is a pedestrian influenced by multiple neighbors?
- Many models, little evidence



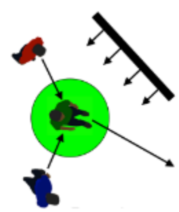
Zonal



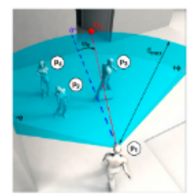
Topological



Alignment



Position-based Force

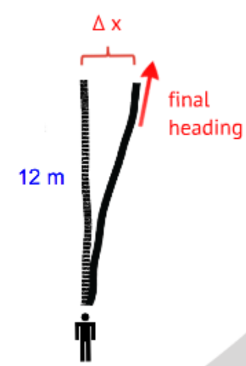


Heuristics

Manipulate Virtual Crowd

perturb heading: ± 10˚

- Participant "walks with" crowd
- Perturb heading or speed of subset S
- Measure lateral deviation or speed change



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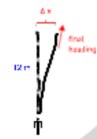
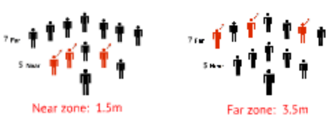


Exp. 1: Superposition

(Rio & Warren, 2014)

- How are multiple neighbors combined?



- Crowd = 12 neighbors
- Vary subset size (S = 0-12)
- Two distance zones
- N=10

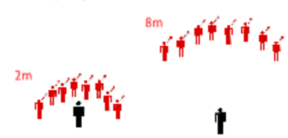
Exp. 2: Metric or Topological Neighborhood?

(Trent Wirth)

- Crowd = 12 neighbors
- Perturb nearest neighbors (S=0,2,4)
- Vary density, hold NN at constant distance
- N=12

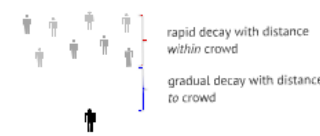
Exp 3: Fixed or Variable Radius?

(Emily Richmond & Trent Wirth)



- Vary crowd distance (2-8m)
- Vary crowd size (2,4,8)
- Perturb all
- N=12

Exp. 4: Double Decay Hypothesis

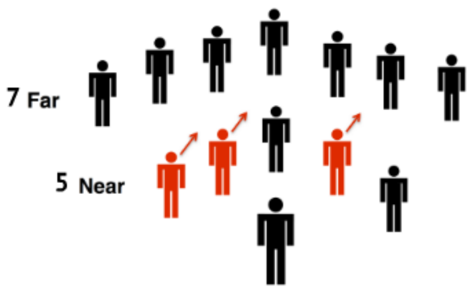


- Neighborhood results from two decay rates
  - variable radius

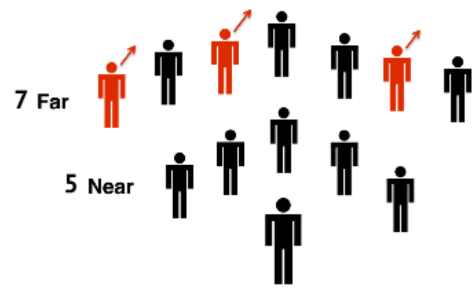
Exp. 1: Superposition

(Rio & Warren, 2014)

- How are multiple neighbors combined?

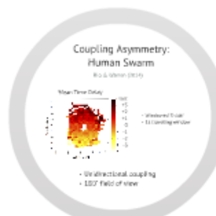


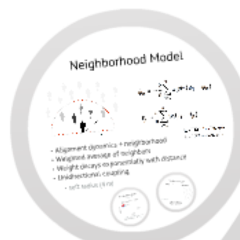
Near zone: 1.5m

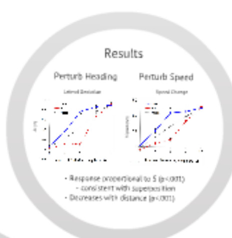


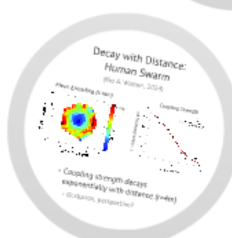
Far zone: 3.5m

- Crowd = 12 neighbors
- Vary subset size (S = 0-12)
- Two distance zones
- N=10



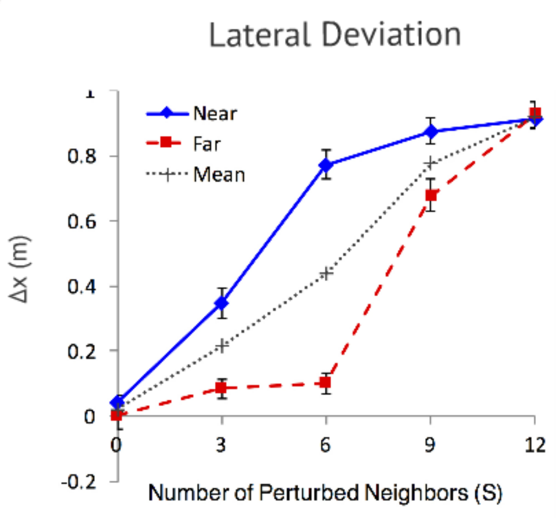




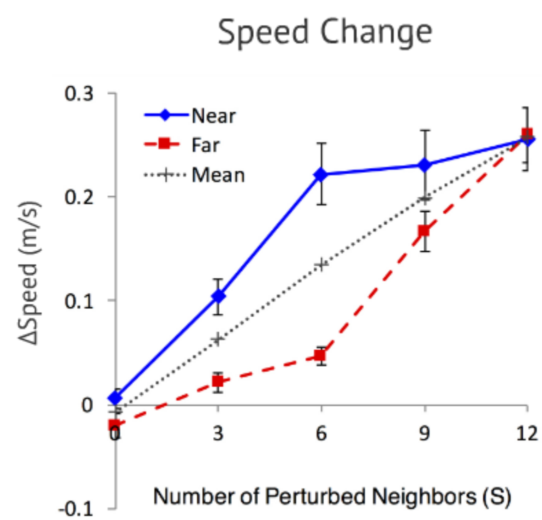


Results

Perturb Heading



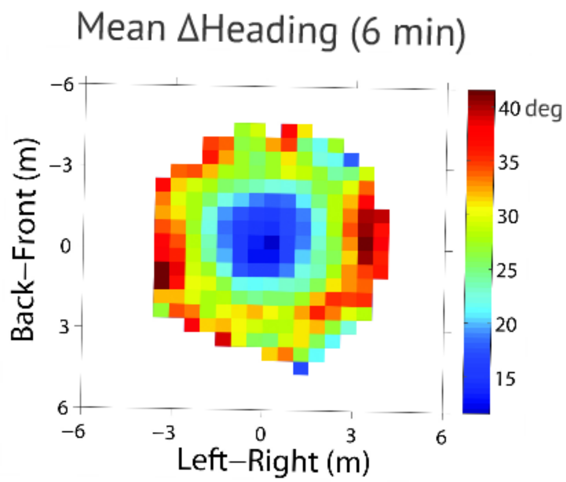
Perturb Speed

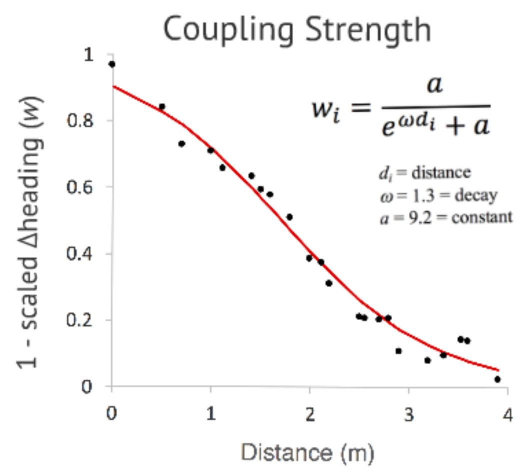


- Response proportional to S (p<.001)
  - consistent with superposition
- Decreases with distance (p<.001)

Decay with Distance: Human Swarm

(Rio & Warren, 2014)



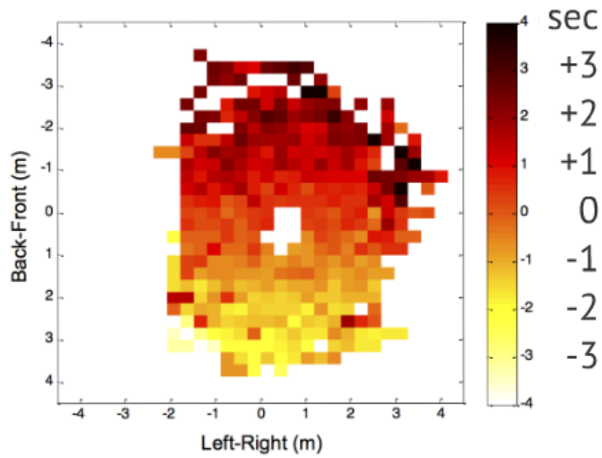


- Coupling strength decays exponentially with distance (r=4m)
  - Occlusion, perspective?

Coupling Asymmetry: Human Swarm

Rio & Warren (2014)

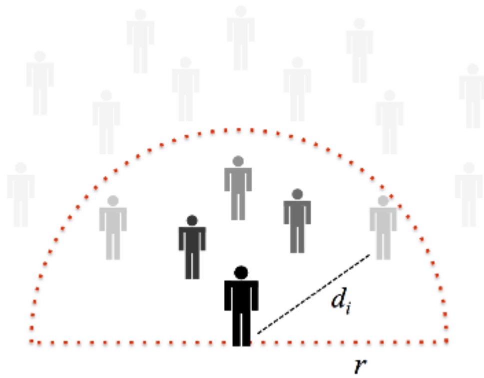
Mean Time Delay



- Windowed X-corr
- 1s traveling window

- Unidirectional coupling
- 180˚ field of view

Neighborhood Model

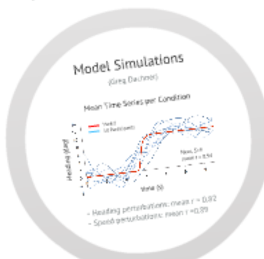


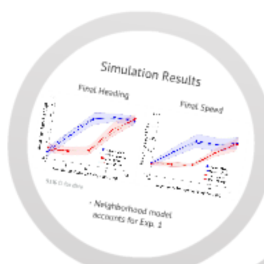
$$\ddot{\phi}_p = -\frac{k}{n}\sum_{i=1}^{n} w_i sin(\phi_i - \phi_p)$$

$$\ddot{r}_p = \frac{c}{n}\sum_{i=1}^{n} w_i(\dot{r}_i - \dot{r}_p)$$

k = 0.81 = heading gain
c = 1.87 = speed gain

- Alignment dynamics + neighborhood
- Weighted average of neighbors
- Weight decays exponentially with distance
- Unidirectional coupling
    - soft radius (4 m)

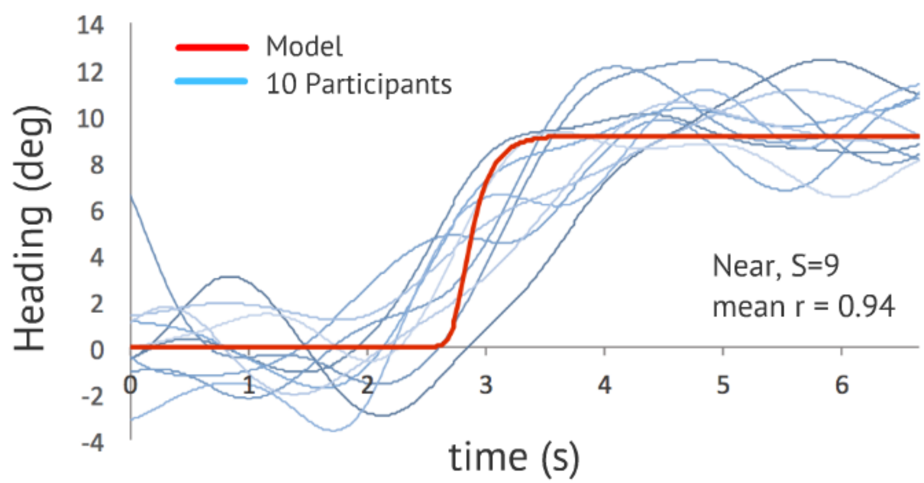




Model Simulations

(Greg Dachner)

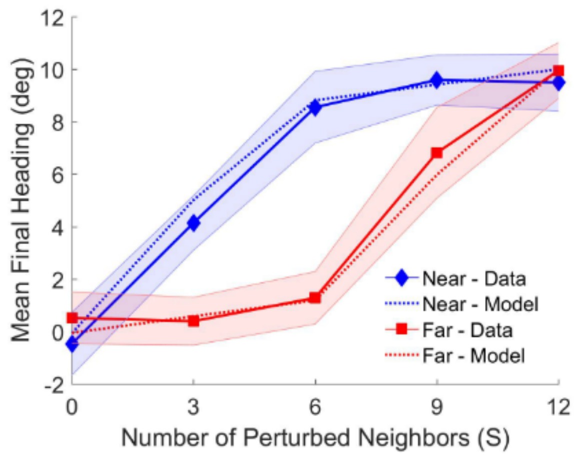
Mean Time Series per Condition



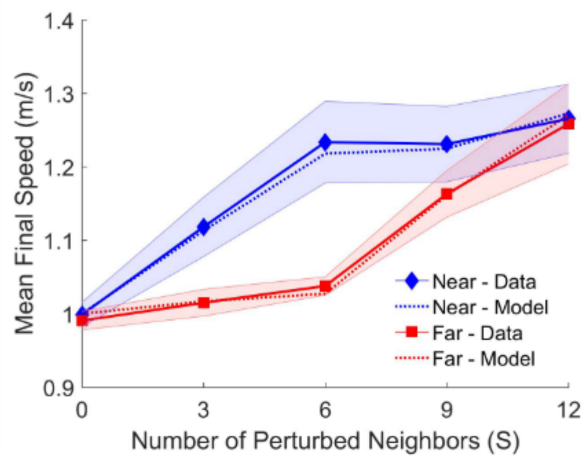
- Heading perturbations: mean r = 0.82
- Speed perturbations: mean r =0.89

Simulation Results

Final Heading



Final Speed

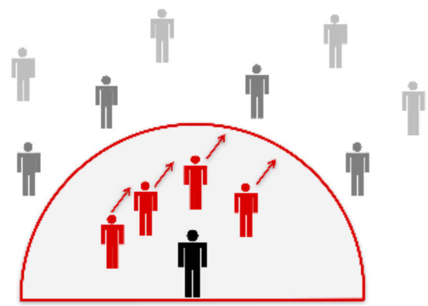


95% CI for data

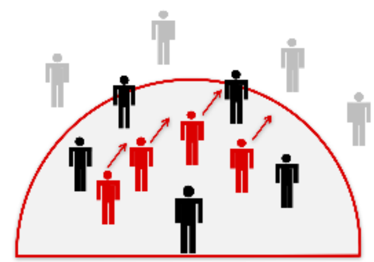
- Neighborhood model accounts for Exp. 1

Exp. 2: Metric or Topological Neighborhood?

(Trent Wirth)

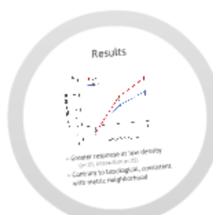


Low Density:
stronger response

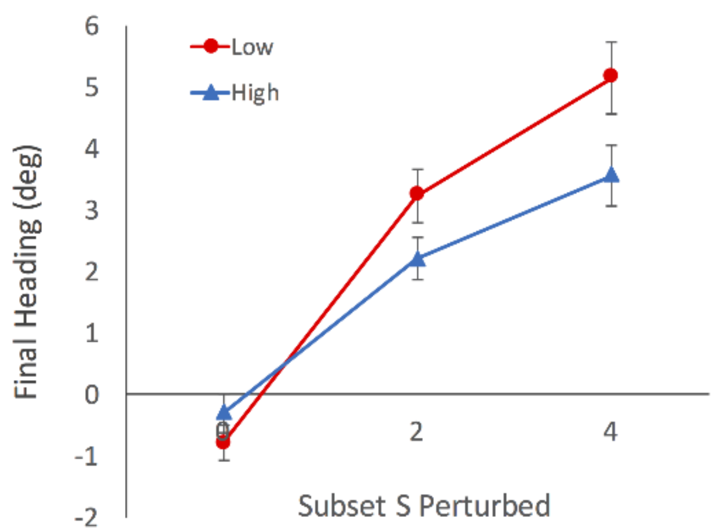


High Density:
weaker response

- Crowd = 12 neighbors
- Perturb nearest neighbors (S=0,2,4)
- Vary density, hold NN at constant distance
- N=12



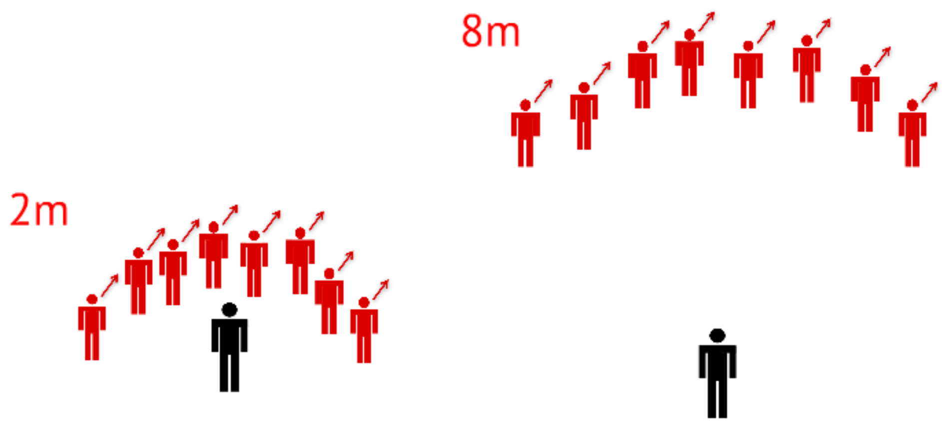
Results



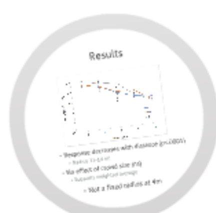
- Greater response at low density
  (p<.05, interaction p<.05)
- Contrary to topological, consistent with metric neighborhood

Exp 3: Fixed or Variable Radius?

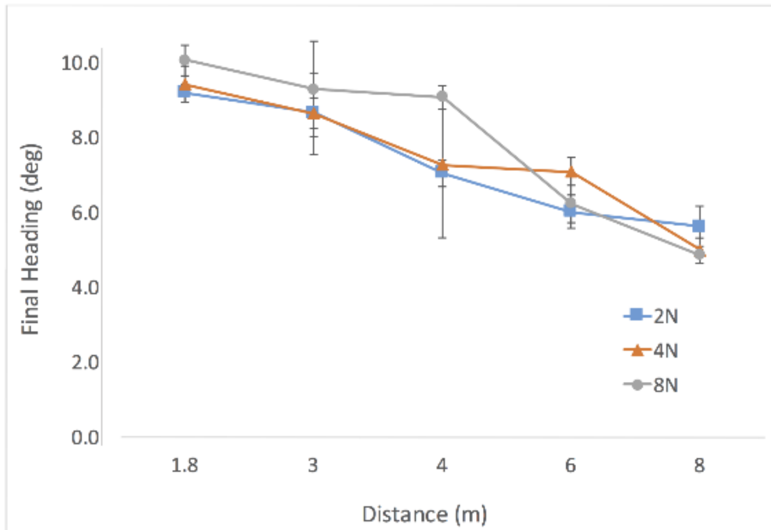
(Emily Richmond & Trent Wirth)



- Vary crowd distance (2-8m)
- Vary crowd size (2,4,8)
- Perturb all
- N=12

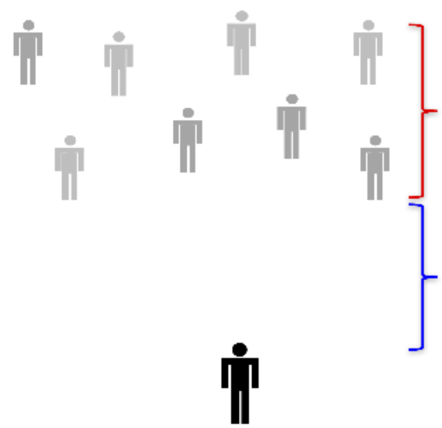


Results



- Response decreases with distance ($p<.0001$)
  - Radius 11-14 m!
- No effect of crowd size (ns)
  - Supports weighted average
- Not a fixed radius at 4m

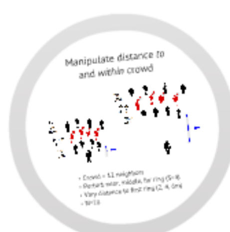
Exp. 4: Double Decay Hypothesis

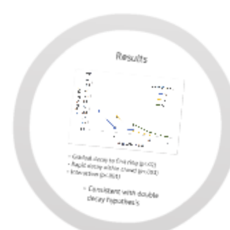


rapid decay with distance *within* crowd

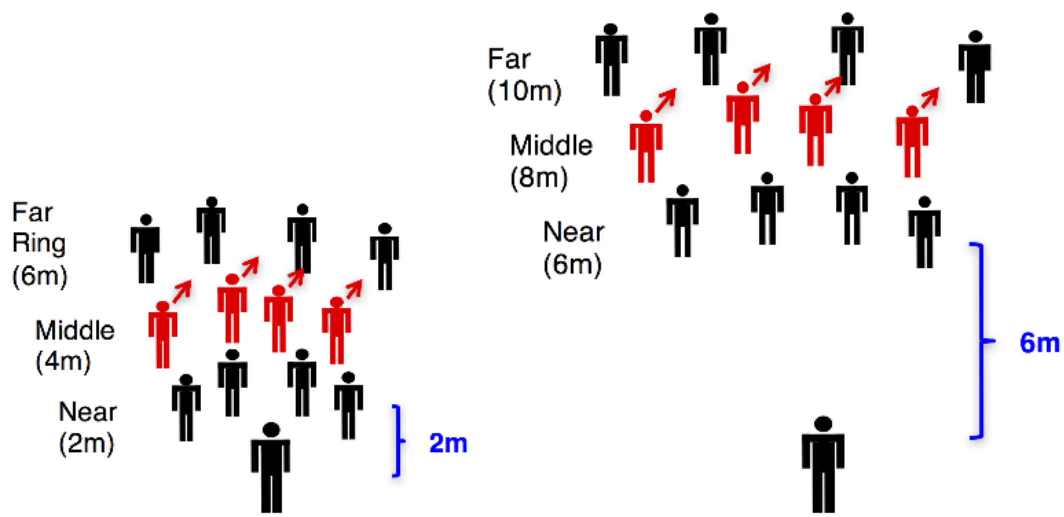
gradual decay with distance *to* crowd

- Neighborhood results from two decay rates
  - variable radius



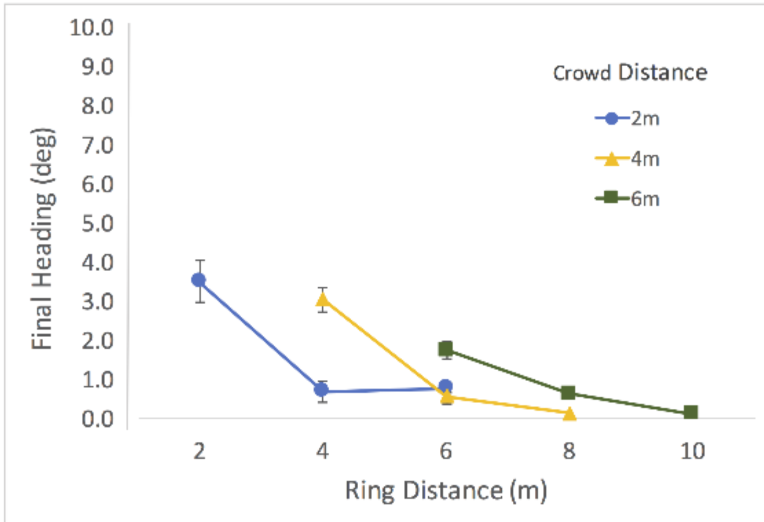


Manipulate distance *to* and *within* crowd



- Crowd = 12 neighbors
- Perturb near, middle, far ring (S=4)
- Vary distance to first ring (2, 4, 6m)
- N=10

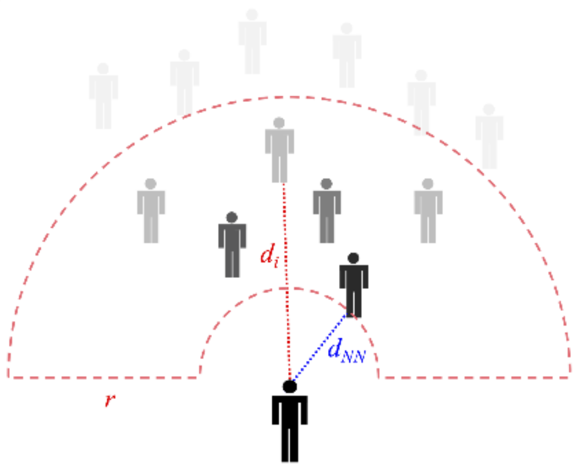
Results



- Gradual decay to first ring ($p<.01$)
- Rapid decay within crowd ($p<.001$)
- Interaction ($p<.001$)

- Consistent with double decay hypothesis

Double-Decay Model



$$\ddot{\phi}_p = -\frac{k}{n}\sum_{i=1}^{n} w_i sin(\phi_i - \phi_p)$$

$$w_i = \left(\frac{a}{e^{\eta(d_i)} + e^{\omega(d_i - d_{NN})} + a}\right)$$

$\eta = 0.4$ decay rate to NN
$\omega = 1.2$ decay rate within crowd
$a = 9.2$ scaling constant

Two exponential decay rates:

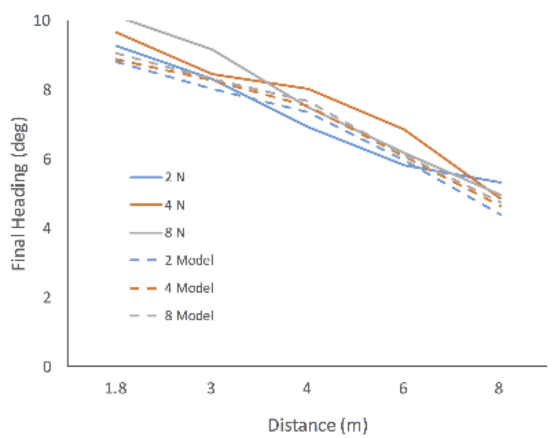
- gradual decay to NN (r=11m): perspective?
- rapid decay within crowd (r=4m): occlusion?
- serves purpose of a topological neighborhood

Conclusio
Neighborhoo

- Superposition
- Metric radius, dou
- Unidirectional cou
- Can model predict motion?

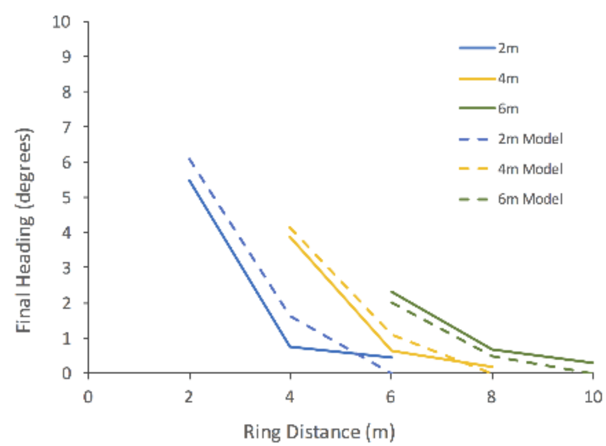
Simulation Results

Exp. 3



- Decay to NN

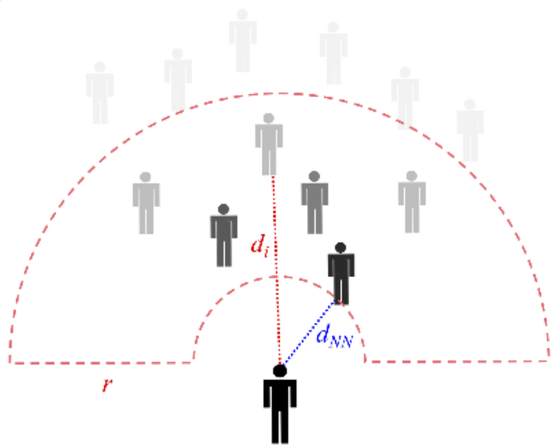
Exp. 4



- Decay to and within crowd

- Double-decay model characterizes neighborhood

Double-Decay Model



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Two exponential decay rates:

- gradual decay to NN (r=11m): perspective?
- rapid decay within crowd (r=4m): occlusion?
- serves purpose of a topological neighborhood

Conclusion 2: Neighborhood Model

- Superposition
- Metric radius, double decay
- Unidirectional coupling
- Can model predict collective motion?

Bottom-up approach
Experiment-driven
Vision-based model

crowd
simulations

Collective Behavior
(global pattern)

neighborhood
model

interactions
generate

visual
neighborhood

positive
feedback

Claims

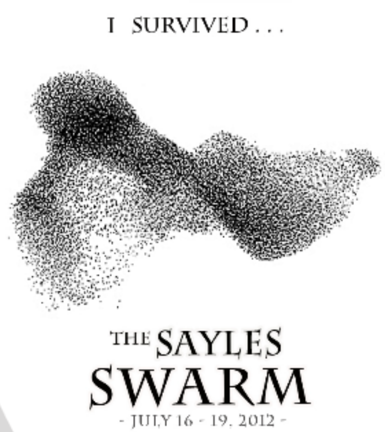
- Bottom-up, experimentally-driven model of crowd dynamics
- Accounts for both local trajectories and global motion
- Collective motion emerges from experimentally-specified local interactions

Individual Behavior
(local coupling)

pedestrian
model

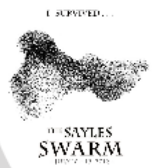
Haken (1977)

The Sayles Swarm



- 16 cameras, 12m x 20m
- N=20, key scenarios

The Sayles Swarm

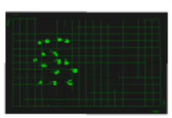


- 16 cameras, 12m x 20m
- N=20, key scenarios

Conclusion 3: Crowd Dynamics

- Alignment dynamics + neighborhood model = local interactions
- Model reproduces individual trajectories and collective motion

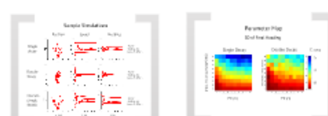
1 Human Swarm



- Veer L/R, stay together, 2 min
- N=16-20, density = 1m, 2m
- Reproduce at local and global levels?

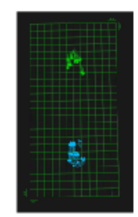
Exploratory Simulations

- Define initial conditions, let go, all agents interact
- Vary initial density, initial heading range
- Compare single and double decay models



- Coherent motion over a wide range of initial conditions
- Wider range with Double Decay than Single Decay model

2 Counterflow

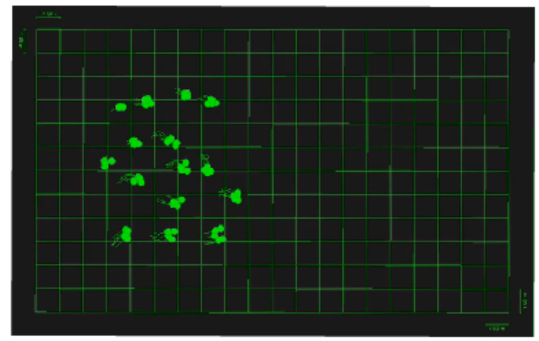


- Two groups, pass through
- Spontaneous lane formation

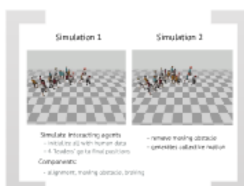
Lanes emerge from following neighbors + avoiding obstacles

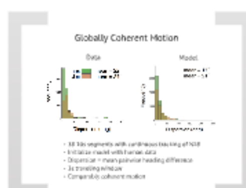
- When is another pedestrian a neighbor or an obstacle?

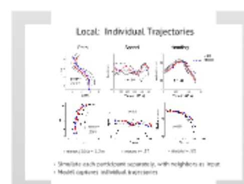
1 Human Swarm



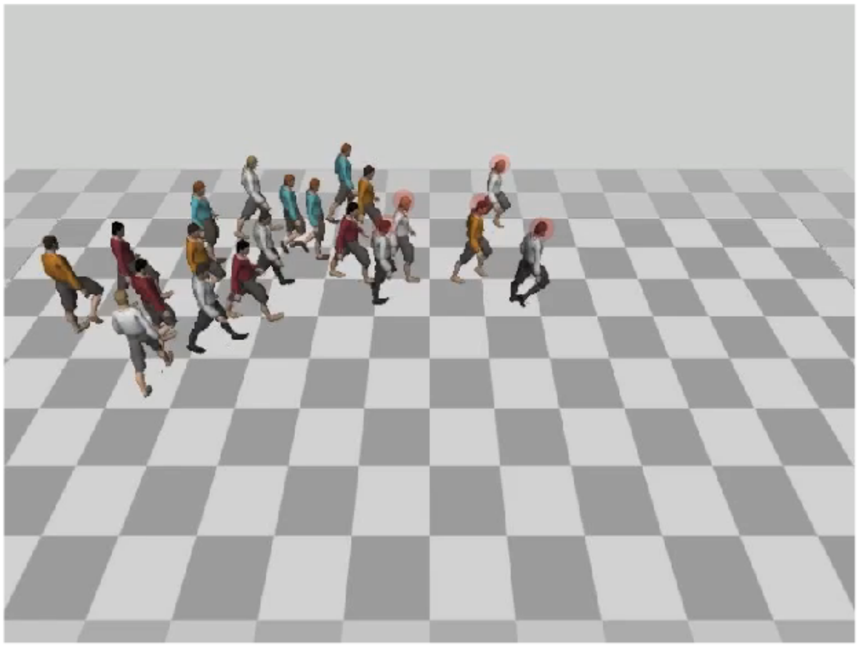
- Veer L/R, stay together, 2 min
- N=16-20, density = 1m, 2m
- Reproduce at local and global levels?







Simulation 1



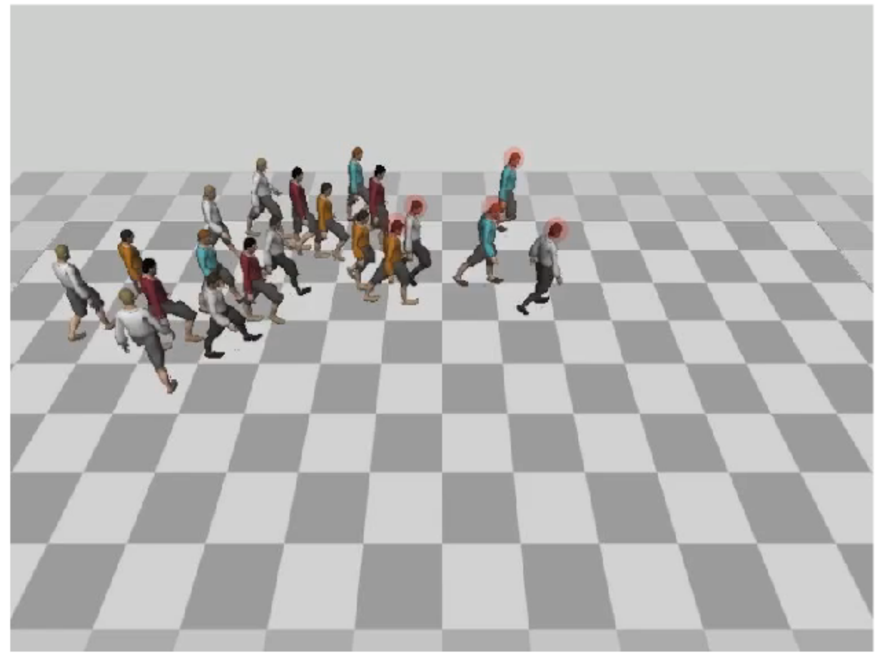
Simulate interacting agents

- initialize all with human data
- 4 'leaders' go to final positions

Components:

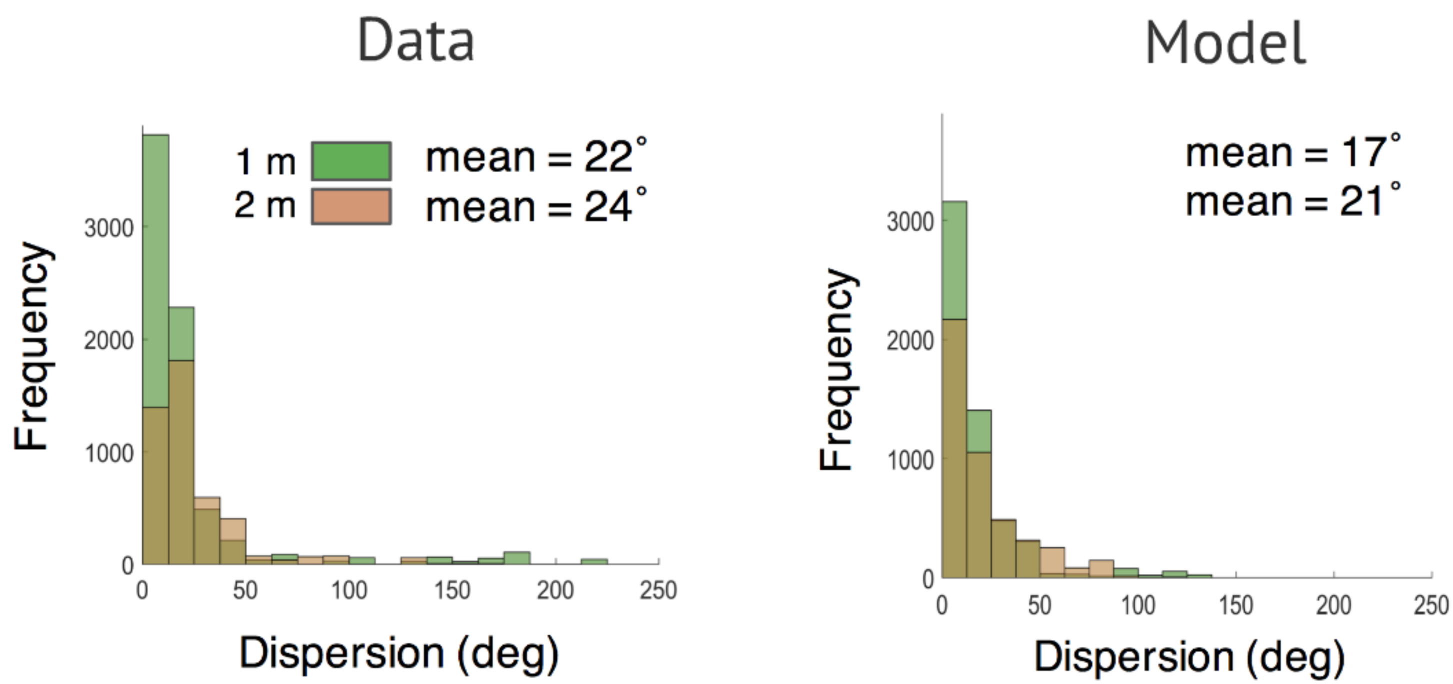
- alignment, moving obstacle, braking

Simulation 2



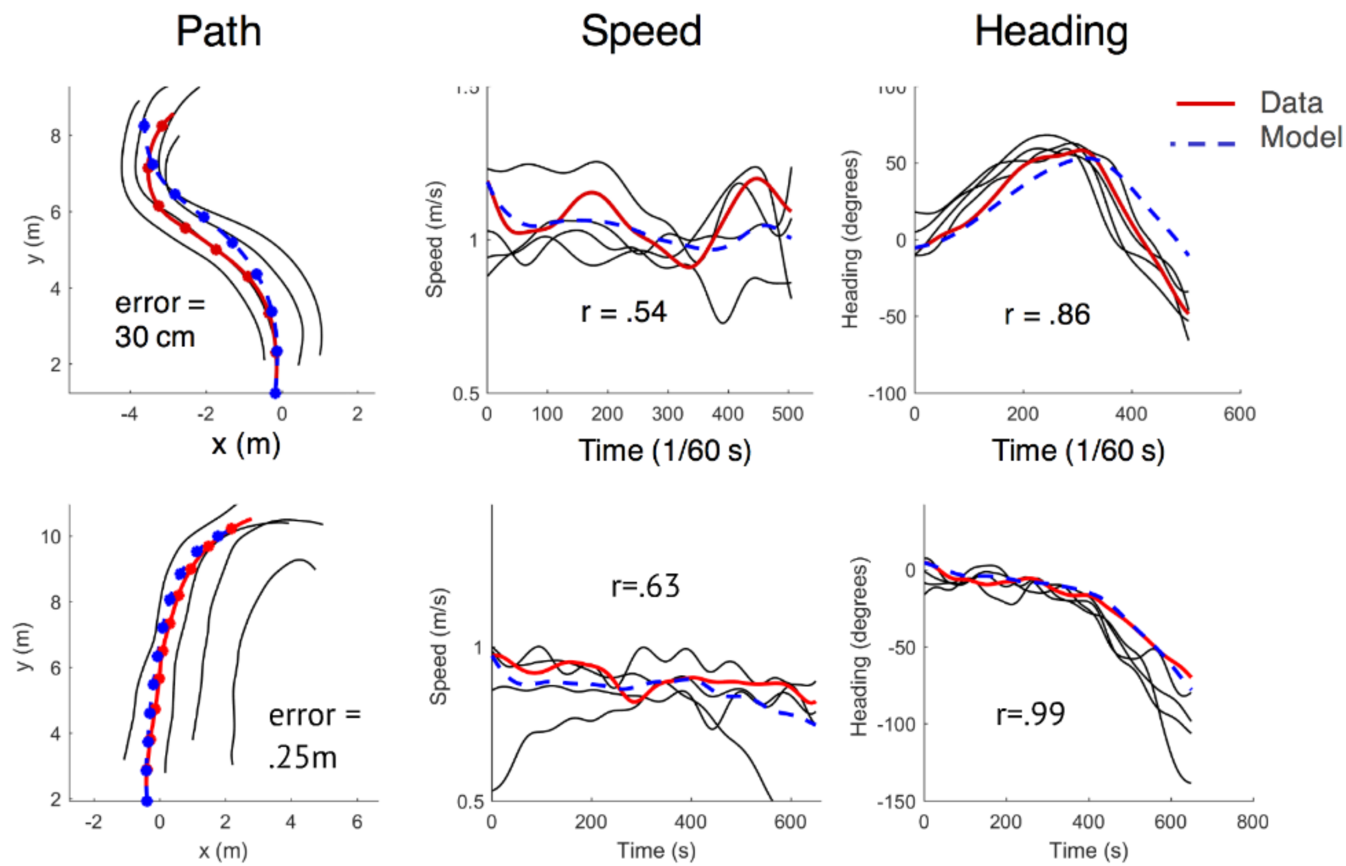
- remove moving obstacle
- generates collective motion

Globally Coherent Motion



- 38 10s segments with continuous tracking of N≥8
- Initialize model with human data
- Dispersion = mean pairwise heading difference
- 3s traveling window
- Comparably coherent motion

Local: Individual Trajectories

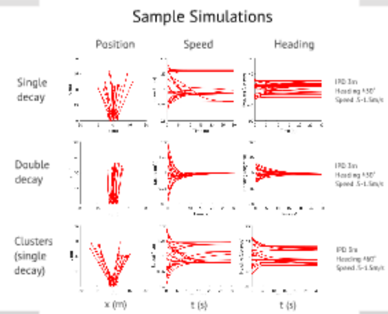


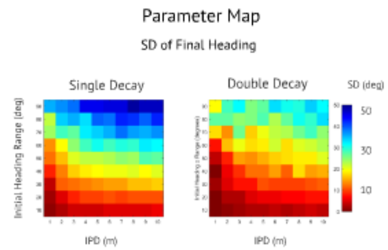
- mean (10s) = 1.2m
- mean r= .57
- mean r= .95

- Simulate each participant separately, with neighbors as input
- Model captures individual trajectories

Exploratory Simulations

- Define initial conditions, let go, all agents interact
- Vary initial density, initial heading range
- Compare single and double decay models

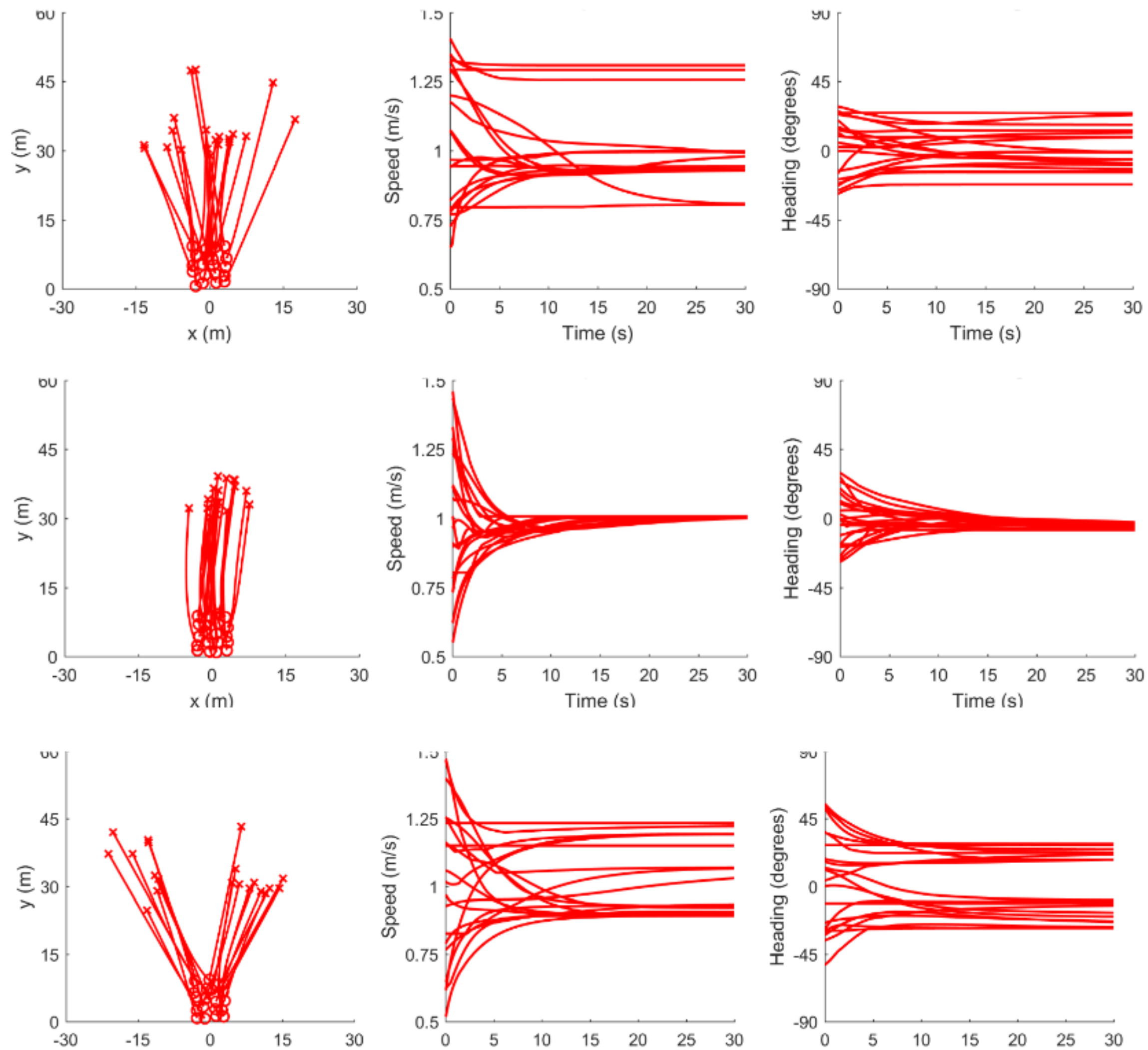




- Coherent motion over a wide range of initial conditions
- Wider range with Double Decay than Single Decay model

Sample Simulations

Position | Speed | Heading



x (m) | t (s) | t (s)

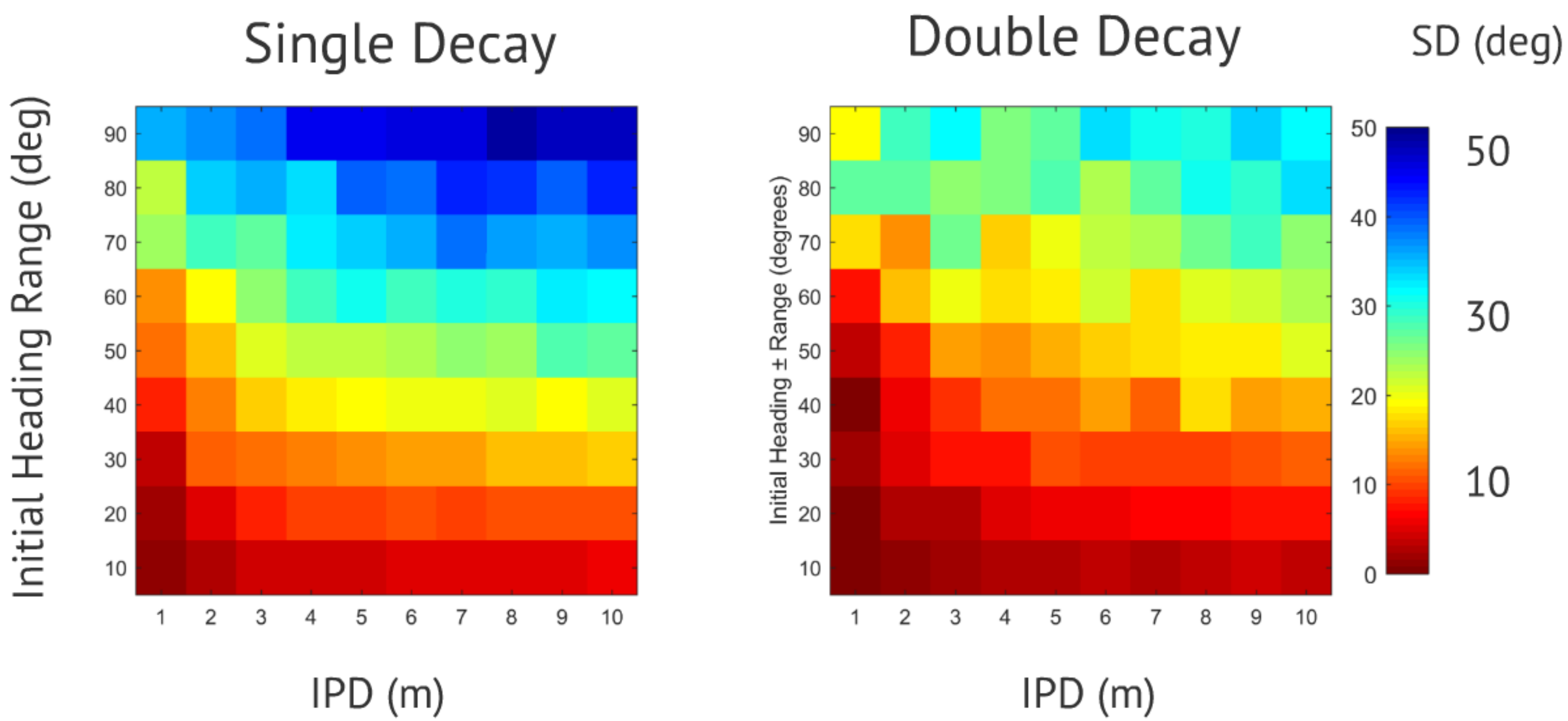
Single decay — IPD 3m, Heading ±30°, Speed .5-1.5m/s

Double decay — IPD 3m, Heading ±30°, Speed .5-1.5m/s

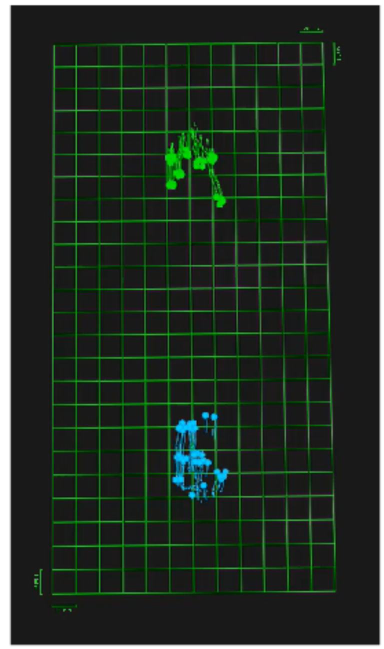
Clusters (single decay) — IPD 3m, Heading ±60°, Speed .5-1.5m/s

Parameter Map

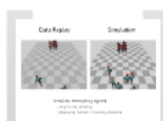
SD of Final Heading



2 Counterflow



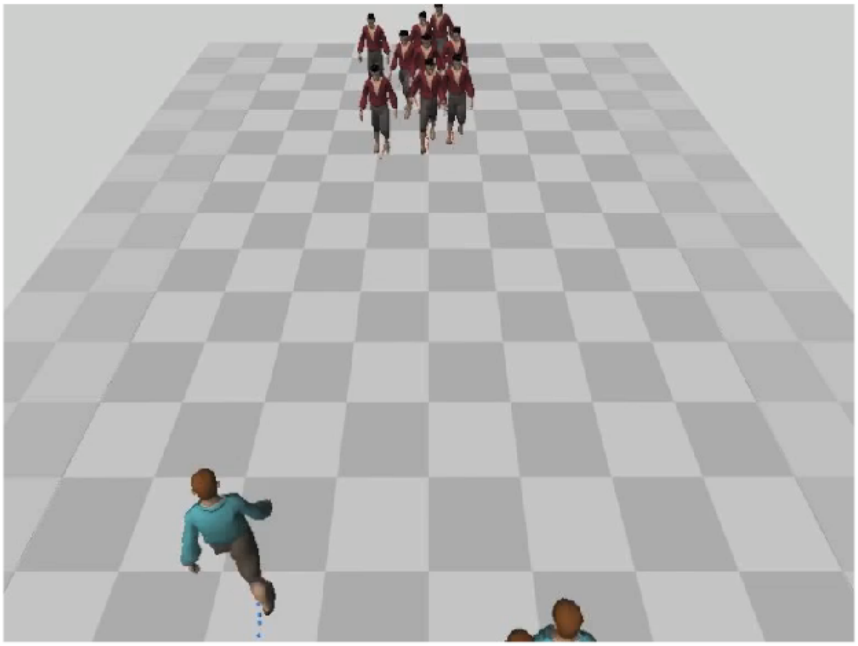
- Two groups, pass through
- Spontaneous lane formation



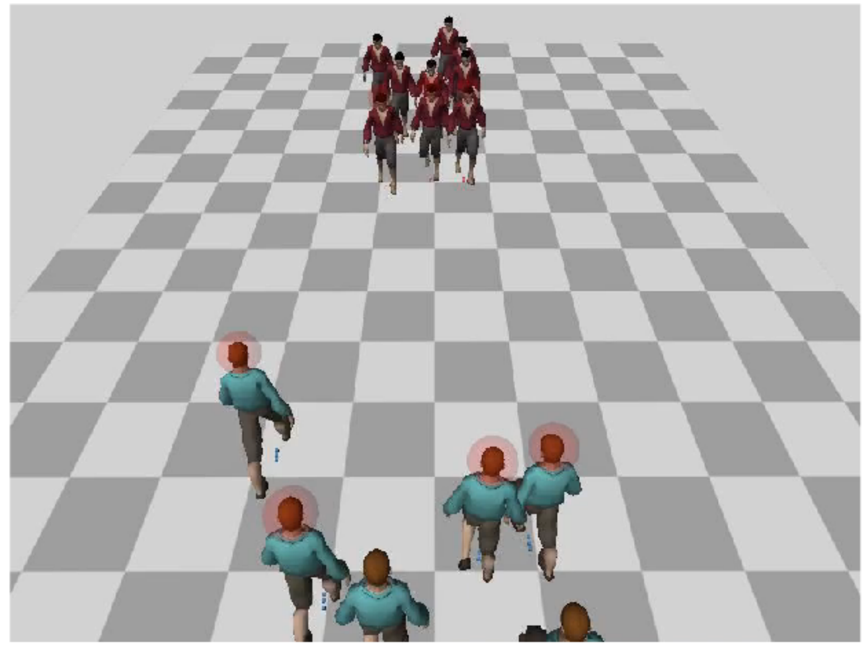
Lanes emerge from following neighbors + avoiding obstacles

- When is another pedestrian a neighbor or an obstacle?

Data Replay



Simulation



Simulate interacting agents

- alignment, braking
- opposing motion = moving obstacle

Bottom-up approach
Experiment-driven
Vision-based model

crowd
simulations

Collective Behavior
(global pattern)

neighborhood
model

interactions
generate

visual
neighborhood

positive
feedback

Individual Behavior
(local coupling)

pedestrian
model

Claims

- Bottom-up, experimentally-driven model of crowd dynamics
- Accounts for both local trajectories and global motion
- Collective motion emerges from experimentally-specified local interactions

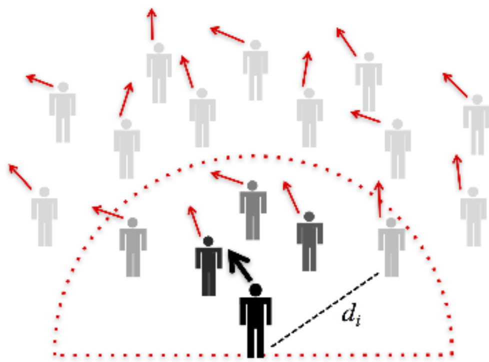
Haken (1977)

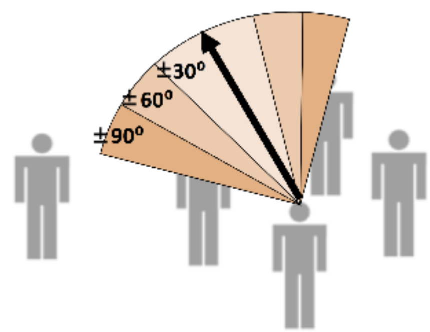
Next

Pattern Formation

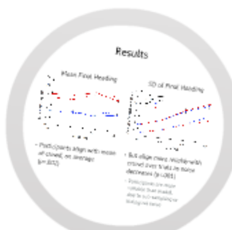
- Phase transition: shoal --> school
- Aligned neighbors recruit more individuals, pattern propagates
- Visual neighborhood as positive feedback?

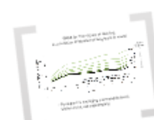
Exp. 5: Noisy Neighbors





- Add noise to neighbor headings (range = ±0˚- 90˚) about mean crowd direction (10˚, 20˚ turn)
- As noise decreases, participant should align more strongly with mean of virtual crowd

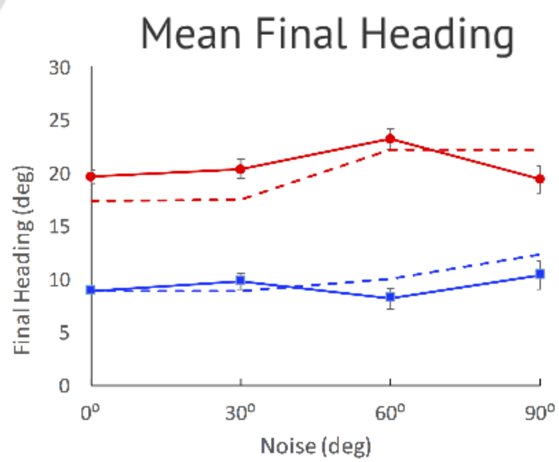


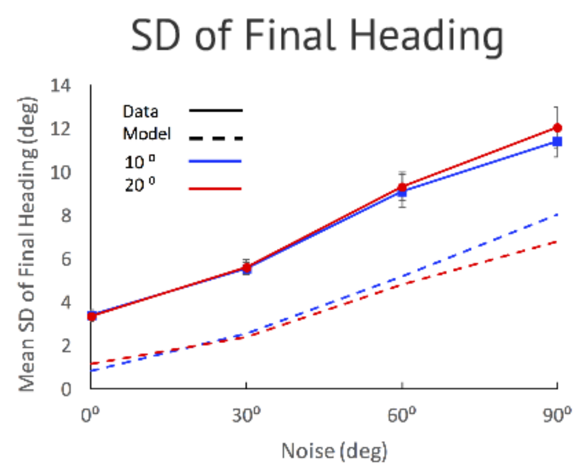


Concl
Positive

- Greater alig
neighborho
stronger pos
- Mechanism
formation

Results

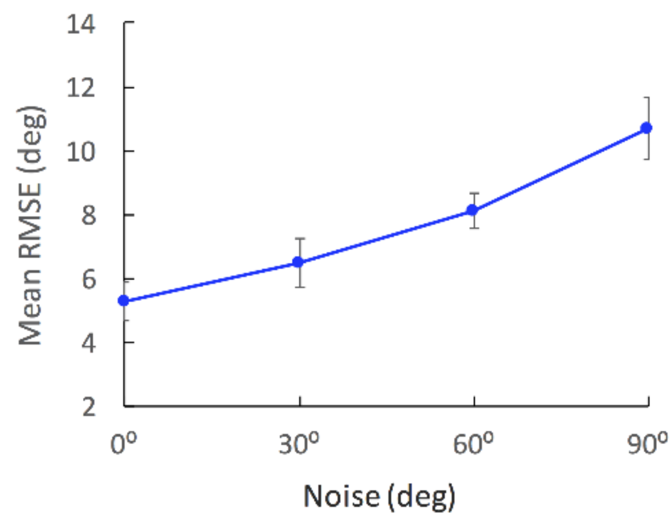




- Participants align with mean of crowd, on average (p<.001)

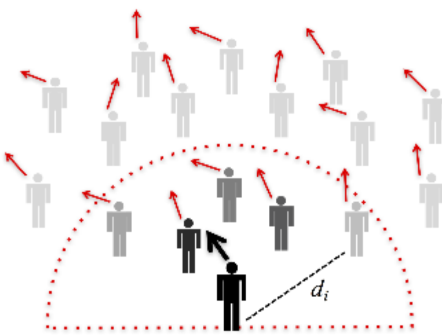
- But align more *reliably* with crowd over trials as noise decreases (p<.001)
  - Participants are more variable than model, due to sub-sampling or biological noise

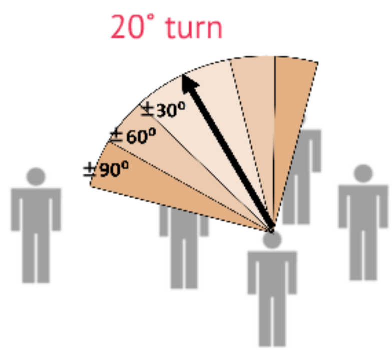
RMSE Between Model and Data
for Time Series of Heading



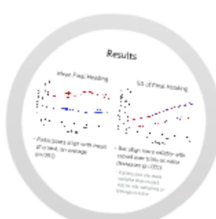
- Participants more closely align with mean of neighborhood (model) *within* a trial as noise decreases (p<.001). Sub-sampling?
- When neighbors are more aligned, the participant is more strongly attracted to the common motion

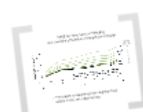
Exp. 5: Noisy Neighbors





- Add noise to neighbor headings (range = ±0°- 90°) about mean crowd direction (10°, 20° turn)
- As noise decreases, participant should align more strongly with mean of virtual crowd





Conclusion 4: Positive Feedback

- Greater alignment within neighborhood creates a stronger positive feedback
- Mechanism of pattern formation

Bottom-up approach
Experiment-driven
Vision-based model

crowd simulations

Collective Behavior
(global pattern)

neighborhood model

interactions generate

visual neighborhood

positive feedback

Individual Behavior
(local coupling)

pedestrian model

Haken (1977)

Claims

- Bottom-up, experimentally-driven model of crowd dynamics
- Accounts for both local trajectories and global motion
- Collective motion emerges from experimentally-specified local interactions

Next

Next

- Extend vision-based model to crowd
- Visual neighborhood as +feedback
  - temporal averaging, sub-sampling
- Network analysis of human swarm
- Generalize model to other scenarios
- Micro --> Mean Field --> Macro